

RATIONAL EXPRESSIONS

Rational Expression: any fraction with variables in the denominator

Simplifying Rational Expressions:

Recall: A fraction is in reduced form if the numerator and denominator have no common factors. The key to simplifying rational expressions is your ability to factor polynomials.

Reduce the following expressions:

Example 1: $\frac{x^2 + 4x - 12}{3x - 6} = \frac{(x+6)(\cancel{x-2})}{3(\cancel{x-2})} = \boxed{\frac{x+6}{3}}$

Example 2: $\frac{16 - x^2}{2x^2 - 9x + 4} = \frac{-x^2 + 16}{2x^2 - 9x + 4} = \frac{-(x^2 - 16)}{(2x-1)(x-4)} = \frac{-(x+4)(\cancel{x-4})}{(2x-1)(\cancel{x-4})}$
 $\rightarrow \boxed{\frac{-x+4}{2x-1}}$

Remember to avoid this common error:

$$\frac{x+3}{3} \neq \frac{x+3}{3} \neq x$$

You can only divide common factors, not terms.

I. Reduce the following rational expressions. Show work.

$$1) \frac{3x+9}{5x+15} = \frac{\cancel{3(x+3)}}{\cancel{5(x+3)}}$$

$$1) \frac{3}{5}$$

$$2) \frac{y^2-25}{2y+10} = \frac{\cancel{(y+5)(y-5)}}{\cancel{2(y+5)}}$$

$$2) \frac{y-5}{2}$$

$$3) \frac{5m}{15m^2-5m} = \frac{\cancel{5m}}{\cancel{5m}(3m-1)}$$

$$3) \frac{1}{3m-1}$$

$$4) \frac{8x-4}{1-4x^2} = \frac{4(2x-1)}{-4x^2+1} = \frac{4(2x-1)}{-(2x+1)(2x-1)}$$

$$4) \frac{4}{-(2x+1)}$$

$$5) \frac{y^2-3y}{y^2-4y+3} = \frac{\cancel{y(y-3)}}{\cancel{(y-1)(y-3)}}$$

$$5) \frac{y}{y-1}$$

$$6) \frac{r^2-9}{r^2-2r-15} = \frac{\cancel{(r+3)(r-3)}}{\cancel{(r+5)(r-3)}}$$

$$6) \frac{\cancel{r+3}}{\cancel{r+5}} = \frac{r-3}{r-5}$$

$$7) \frac{5x^2+10xy}{15xy+30y^2} = \frac{\cancel{5x(x+2y)}}{\cancel{15y(x+2y)}} = \frac{x}{3y}$$

$$7) \frac{x}{3y}$$

$$8) \frac{a^2-b^2}{b-a} = \frac{(a+b)(a-b)}{-a+b} = \frac{(a+b)(a-b)}{-(a-b)}$$

$$8) \frac{-(a+b) \text{ or } -a-b}{1}$$

II. Multiplying Rational Expressions:

Both rational numbers and rational expressions are multiplied in the same way:

$$\frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd} \quad \text{where } b \text{ and } d \neq 0$$

Always express answers in simplest form.

Example 1: $\frac{\cancel{4}^1 \cdot \cancel{15}^3}{\cancel{5}^1 \cdot \cancel{16}^4} = \boxed{\frac{3}{4}}$

Example 2: $\frac{\cancel{x}^2}{\cancel{x}+3} \cdot \frac{(x+2)(\cancel{x}+3)}{2\cancel{x}} = \boxed{\frac{x^2(x+2)}{2}}$

Sometimes you will need to factor before you multiply in order to find the common factors so you can reduce the expression.

Example 3: $\frac{x^2-9}{x^2+x-12} \cdot \frac{x+2}{x+3} = \frac{(\cancel{x+3})(\cancel{x-3})}{(x+4)(\cancel{x-3})} \cdot \frac{x+2}{(\cancel{x+3})} = \boxed{\frac{x+2}{x+4}}$

Example 4: $\frac{x^3+5x^2}{(x+5)^2} \cdot \frac{x^2-25}{x^2} = \frac{\cancel{x}^2(\cancel{x+5})}{(\cancel{x+5})(\cancel{x+5})} \cdot \frac{(\cancel{x+5})(x-5)}{\cancel{x^2}} = \boxed{x-5}$

II. Multiply and simplify. Show work.

$$1) \frac{5x+15}{4x+8} \cdot \frac{2x+4}{25} = \frac{\cancel{5}(x+3)}{\cancel{4}(x+2)} \cdot \frac{\cancel{2}(x+2)}{\cancel{25}^5}$$

$$1) \frac{x+3}{10}$$

$$2) \frac{x^2-16}{x+3} \cdot \frac{7x+21}{2x+8} = \frac{(x+4)(x-4)}{\cancel{x+3}} \cdot \frac{7(x+3)}{2(x+4)}$$

$$2) \frac{7(x-4)}{2}$$

$$3) \frac{3x^2-15x}{x^2-25} \cdot \frac{2x+10}{18x} = \frac{\cancel{3x}(x-5)}{(\cancel{x+5})(x-5)} \cdot \frac{2(x+5)}{18x}$$

$$3) \frac{1}{3}$$

$$4) \frac{x^2-5x}{x^2-4} \cdot \frac{3x+6}{x^3-5x^2} = \frac{\cancel{x}(x-5)}{(\cancel{x+2})(x-2)} \cdot \frac{3(\cancel{x+2})}{x^2(x-5)}$$

$$4) \frac{3}{x(x-2)}$$

$$5) \frac{x^2-4x+4}{9} \cdot \frac{3x+6}{x^2-4} = \frac{(x-2)(x-2)}{\cancel{9}^3} \cdot \frac{3(x+2)}{(\cancel{x+2})(x-2)}$$

$$5) \frac{x-2}{3}$$

$$6) \frac{x^2-2x+1}{7x-7} \cdot \frac{21}{x^2-3x+2} = \frac{(x-1)(x-1)}{\cancel{7}(x-1)} \cdot \frac{\cancel{21}^3}{(x-2)(x-1)}$$

$$6) \frac{3}{x-2}$$

$$7) \frac{18}{x^2+3x+2} \cdot \frac{x^2-2x-8}{3x-12} = \frac{\cancel{18}^6}{(\cancel{x+2})(x+1)} \cdot \frac{(x-4)(x+2)}{3(x-4)}$$

$$7) \frac{6}{x+1}$$

$$8) \frac{x^2+x-20}{2x+10} \cdot \frac{16}{x^2-3x-4} = \frac{(\cancel{x+5})(x-4)}{2(\cancel{x+5})} \cdot \frac{\cancel{16}^8}{(\cancel{x-4})(x+1)}$$

$$8) \frac{8}{x+1}$$

$$9) \frac{64-x^2}{3x^2+x} \cdot \frac{3x}{x^2-64} = \frac{-(x+8)(x-8)}{\cancel{3x}(x+1)} \cdot \frac{\cancel{3x}}{(\cancel{x+8})(x-8)}$$

$$9) \frac{-3}{3x+1}$$

$$10) \frac{4x^2-9}{3x} \cdot \frac{9x^2-6x}{9-4x^2} = \frac{(\cancel{2x+3})(2x-3)}{\cancel{3x}} \cdot \frac{3x(3x-2)}{-(\cancel{2x+3})(2x-3)}$$

$$10) \frac{-(3x-2)}{-3x+2} \text{ or } \frac{-(3x-2)}{-3x+2}$$

DIVIDING RATIONAL EXPRESSIONS

Divide:

- 1) $\frac{3}{5} \div \frac{7}{2} =$
- 2) $\frac{3}{5} \div \frac{4}{10} =$

Both rational numbers and rational expressions are divided the same way:

$$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c} = \frac{ad}{bc} \quad \text{where } b, c, \text{ and } d \neq 0$$

Never mind the reason why, just ...

Example 1: $\frac{5x}{4y} \div \frac{2x}{3y^2} = \frac{\cancel{5x}}{4\cancel{y}} \cdot \frac{3\cancel{y}^2}{2\cancel{y}} = \boxed{\frac{15y}{8}}$

Example 2: $\frac{x}{x+3} \div \frac{2x}{(x+2)(x+3)} = \frac{\cancel{x}}{\cancel{x+3}} \cdot \frac{(x+2)\cancel{(x+3)}}{2\cancel{x}} = \boxed{\frac{x+2}{2}}$

Example 3: $\frac{x^2-4}{x+3} \div \frac{x^2-4x+4}{x^2+3x} = \frac{(x+2)\cancel{(x-2)}}{\cancel{x+3}} \cdot \frac{x\cancel{(x+3)}}{(x-2)\cancel{(x+2)}} = \boxed{\frac{x(x+2)}{x-2}}$

Example 4: $\frac{x^2-4}{\frac{2}{2-x}} = \frac{(x+2)\cancel{(x-2)}}{\cancel{2}} \cdot \frac{\cancel{4}^2}{\cancel{2-x}^2} = \boxed{-2(x+2)}$

ALGEBRA 2
WORKSHEET 8.4 #2

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Divide and simplify each of the following expressions.

$$1) \quad \frac{x^2 - 25}{2x} \div \frac{3x + 15}{12x^2 - 24x} = \frac{(x-5)(x+5)}{2x} \cdot \frac{2x(x-2)}{2x(x+5)} = \boxed{2(x-5)(x-2)}$$

$$2) \quad \frac{5x - 15}{x^2 - 25} \div \frac{2x}{x^2 + 5x} = \frac{5(x-3)}{(x+5)(x-5)} \cdot \frac{x(x+5)}{2x} = \boxed{\frac{5(x-3)}{2(x-5)}}$$

$$3) \quad \frac{x^2 - 7x + 10}{x - 1} \div \frac{3x^2 - 6x}{x^2 - 1} = \frac{(x-5)(x-2)}{x-1} \cdot \frac{(x+1)(x-1)}{3x(x-2)} = \boxed{\frac{(x-5)(x+1)}{3x}}$$

$$4) \quad \frac{x^2 - 7x + 6}{x + 3} \div \frac{x^2 - 5x - 6}{x + 3} = \frac{(x-6)(x-1)}{x+3} \cdot \frac{x+3}{(x-6)(x+1)} = \boxed{\frac{x-1}{x+1}}$$

$$5) \quad \frac{x^2 - 2x - 8}{4x^2 + 8x} \div (x^2 + x - 20) = \frac{(x-4)(x+2)}{4x(x+2)} \cdot \frac{1}{(x+5)(x-4)} = \boxed{\frac{1}{4x(x+5)}}$$

$$6) \frac{9x^2 - y^2}{2x} \div \frac{3x^2y + xy^2}{4x^2y^2} = \frac{(3x+y)(3x-y)}{2x} \cdot \frac{4x^2y^2}{xy(3x+y)} = \boxed{2y(3x-y)}$$

$$7) \frac{(m-3)^2}{m^2 - 6m + 9} \div \frac{m^3 - 9m}{m^2 - 9} = \frac{(m-3)(m-3)}{(m-3)(m-3)} \cdot \frac{(m+3)(m-3)}{m(m+3)(m-3)} = \boxed{\frac{1}{m}}$$

$$8) \frac{\frac{x^2 - 4}{x+3}}{\frac{x^2 - 4x + 4}{x^2 + 3x}} = \frac{(x+2)(x-2)}{x+3} \cdot \frac{x(x+3)}{(x-2)(x-2)} = \boxed{\frac{x(x+2)}{x-2}}$$

$$9) \frac{2x^2 + 9x + 9}{x+1} \div \frac{10x^2 + 19x + 6}{5x^2 + 7x + 2} = \frac{(2x+3)(x+3)}{x+1} \cdot \frac{(5x+2)(x+1)}{(2x+3)(5x+2)}$$

~~$\frac{(5x+2)(x+3)}{(x+1)(5x+2)}$~~

$\boxed{x+3}$

$$10) \frac{b^2 - 100}{b^3} \div \frac{3b^2 - 31b + 10}{2b} = \frac{(b+10)(b-10)}{b^3} \cdot \frac{2b}{(3b-1)(b-10)}$$

$$\boxed{\frac{2(b+10)}{b^2(3b-1)}}$$

ALGEBRA 2
WORKSHEET 8.4 #3

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Simplify each rational expression.

$$1) \frac{x^2 + 4x - 5}{18} \cdot \frac{6}{x^2 - x} = \frac{(x+5)(x-1)}{18 \cdot 3} \cdot \frac{6}{x(x-1)} \quad 1) \frac{x+5}{3x}$$

$$2) \frac{x+5}{2x-8} \cdot \frac{5x^2 - 5x - 30}{10x^2 + 70x + 100} = \frac{x+5}{2(x-4)} \cdot \frac{5(x-1)(x-3)}{10(x+5)(x+2)} \quad 2) \frac{(x-3)}{4(x-4)}$$

$$3) \frac{14x^2 - 7x - 21}{14x - 28} \div \frac{x+1}{3x-6} = \frac{7(2x-3)(x+1)}{14(x-2)} \cdot \frac{3(x-2)}{(x+1)} \quad 3) \frac{3(2x-3)}{2}$$

$$4) \frac{x^{12}}{5} \cdot \frac{15}{x^4} \cdot \frac{x^3}{9} = \frac{15x^{15}}{45x^4} = \frac{x^{11}}{3} \quad 4) \frac{x^{11}}{3}$$

$$5) \frac{x^2 + 5x + 6}{x^2 + 2x - 3} \div \frac{10x + 20}{10} = \frac{(x+3)(x+2)}{(x+3)(x-1)} \cdot \frac{10}{10(x+2)} \quad 5) \frac{1}{x-1}$$

$$6) \frac{6x^2 + 30x}{x^2 + 6x + 5} \div \frac{x^2 + 4x + 4}{x^2 - x - 6} = \frac{6x(x+5)}{(x+5)(x+1)} \cdot \frac{(x-3)(x+2)}{(x+2)(x+2)} \quad 6) \frac{6x(x-3)}{(x+2)(x+1)}$$

$$7) \frac{x^2 - 9}{x^2 + 2x - 8} \cdot \frac{x^2 + 9x + 20}{x^2 - 3x} = \frac{(x-3)(x+3)}{(x+4)(x-2)} \cdot \frac{(x+5)(x+4)}{x(x-3)} \quad 7) \frac{(x+3)(x+5)}{x(x-2)}$$

$$8) \frac{4x^2 + 27x - 7}{9x^2 + 12x - 5} \cdot \frac{4x^2 - 7x + 3}{2x^2 + 13x - 7} = \frac{(4x+7)(x-1)}{(3x-1)(3x+5)} \cdot \frac{(4x-3)(x-1)}{(2x-1)(x+7)} \quad 8) \frac{(4x+7)(x-1)}{(3x-1)(3x+5)}$$

$$9) \frac{14x^3}{27} \cdot \frac{-9}{2x^5} \div \frac{3}{-6x^3} = \frac{14x^3}{27} \cdot \frac{-9}{2x^5} \cdot \frac{-1}{-2} = \frac{14x}{3} \cdot \frac{-9}{2} \cdot \frac{-1}{2}$$

$$9) \frac{14x}{3}$$

$$10) \frac{x^4 - 81}{3x^2 + 27} \div \frac{x^2 - x - 12}{3x} = \frac{(x^2+9)(x^2-9)}{3(x^2+9)} \cdot \frac{3x}{(x+4)(x+3)} = \frac{x(x+3)(x-3)}{(x+4)(x+3)}$$

$$10) \frac{x(x-3)}{(x+4)}$$

$$11) \frac{4x-8}{x^2-x-6} \cdot \frac{x^2-9}{x^3+x^2-6x} = \frac{4(x-2)}{(x-3)(x+2)} \cdot \frac{(x+3)(x-3)}{x(x+3)(x-2)} = \frac{4}{x(x+2)}$$

$$11) \frac{4}{x(x+2)}$$

$$12) \frac{x^2+6x+9}{x^2+10x+24} \cdot \frac{x^2+3x-18}{x+3} = \frac{(x+3)(x+3)}{(x+6)(x+4)} \cdot \frac{(x+3)(x-3)}{(x+3)} = \frac{(x+3)(x-3)}{(x+4)}$$

$$12) \frac{(x+3)(x-3)}{(x+4)}$$

$$13) \frac{3x^3+24x^2}{x^2+18x+80} \div \frac{x-9}{x^2+20x+100} = \frac{3x^2(x+8)}{(x+10)(x+8)} \cdot \frac{(x+10)(x+10)}{x-9} = \frac{3x^2(x+10)}{(x-9)}$$

$$13) \frac{3x^2(x+10)}{(x-9)}$$

$$14) \frac{\frac{x^2-y^2}{5x^3y^2}}{\frac{4x+4y}{15x^2y^5}} = \frac{(x+y)(x-y)}{5x^3y^2} \cdot \frac{15x^2y^5}{4(x+y)} = \frac{3y^3(x-y)}{4x}$$

$$14) \frac{3y^3(x-y)}{4x}$$

$$15) \frac{x^2-5x+6}{x^2-8x+15} \cdot \frac{x-5}{x-2} \div \frac{x^2-9}{x^2+3x} =$$

$$15) \frac{x}{(x-3)}$$

$$\frac{(x-3)(x-2)}{(x-3)(x-5)} \cdot \frac{x-5}{x-2} \cdot \frac{x(x+3)}{(x-3)(x+3)}$$

ALGEBRA 2
REVIEW 8.4

NAME _____
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I. Simplify each rational expression. SHOW WORK!

$$1) \frac{x^2 + 8x - 20}{x^2 - 4} = \frac{(x+10)(\cancel{x-2})}{(x+2)(\cancel{x-2})} \quad 1) \frac{x+10}{x+2}$$

$$2) \frac{2x^3 - 12x^2}{x^2 - 3x - 18} = \frac{2x^2(\cancel{x-6})}{(\cancel{x-6})(x+3)} \quad 2) \frac{2x^2}{x+3}$$

$$3) \frac{3x-9}{3-x} = \frac{3(\cancel{x-3})}{-1(\cancel{x-3})} = -3 \quad 3) -3$$

$$4) \frac{x^2 + 5x}{2x^2 + 10x} = \frac{\cancel{x}(x+5)}{2\cancel{x}(x+5)} \quad 4) \frac{1}{2}$$

II. Multiply or Divide as indicated. Give all answers in simplest form.
SHOW WORK!

$$1) \frac{3}{x^2 - 3x} \cdot \frac{x^2 - x - 6}{3x + 6} = \frac{3}{x(\cancel{x-3})} \cdot \frac{(\cancel{x-3})(x+2)}{3(x+2)} \quad 1) \frac{1}{x}$$

$$2) \frac{x^2 + 2x - 15}{x^2 - 9} \cdot \frac{6x + 18}{x + 2} = \frac{(x+5)(\cancel{x-3})}{(\cancel{x+3})(\cancel{x-3})} \cdot \frac{6(\cancel{x+3})}{(x+2)} \quad 2) \frac{6(x+5)}{x+2}$$

$$3) \frac{4-x}{5x} \cdot \frac{x^2+5x}{x^2+x-20} =$$

$$\frac{\cancel{(x+4)}}{5\cancel{x}} \cdot \frac{\cancel{x}(x+5)}{(x+5)\cancel{(x-4)}} =$$

$$3) \underline{-\frac{1}{5}}$$

$$4) \frac{x^2-4x+4}{x^2-1} \div \frac{x-2}{x+1} =$$

$$\frac{(x-2)\cancel{(x-2)}}{(\cancel{x+1})(x-1)} \cdot \frac{(x+1)}{\cancel{(x-2)}} =$$

$$4) \underline{\frac{x-2}{x-1}}$$

$$5) \frac{x^2-xy}{3x} \div \frac{x-y}{15x^2} =$$

$$\frac{\cancel{x}(x-y)}{3\cancel{x}} \cdot \frac{15x^2}{\cancel{(x-y)}} =$$

$$5) \underline{5x^2}$$

$$6) \frac{\frac{x^2}{x^2-7x}}{1} =$$

$$\frac{x^2}{x^2-4x-21}$$

$$\frac{\cancel{x^2}}{\cancel{x}(x-7)} \cdot \frac{\cancel{(x-7)}(x+3)}{1} =$$

$$6) \underline{x(x+3)}$$

